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On the importance of metrics in practical applications

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Abstract

Students motivation for learning mathematical concepts can be increased when showing the usefulness of these concepts in practical problems. One important mathematical concept is the concept of metric space and, more related to the applications, the concept of metric function. In this work we aim to illustrate how important is to appropriately choose the metric when dealing with a practical problem. In particular, we focus on the problem of detection of noisy pixels in colour images. In this context, it is very important to appropriately measure the distances and similarities between the image pixels, which is done by means of an appropriate metric. We study the performance of different metrics, including recent fuzzy metrics, within a specific filter to show that it is indeed a critical choice to appropriately solve the task.

Keywords: Colour image filter, Fuzzy metric, Impulse noise.

1 Introduction

Nowadays, the process of digital signals and images, and particularly colour image processing, is a practical problem extensively studied. A problem that appears during the acquisition and transmission of digital images is impulsive noise, that affects to some pixels of the image, and the reduction of impulsive noise has been extensively studied in the last years. Vector median-based filters [1]-[3] are widely used methods for impulse noise reduction in colour and multichannel images because they are based on the theory of robust statistics and, consequently, perform robustly. These methods apply the filtering operation over all the pixels of the image, and they tend to blur details and edges of the image.

To overcome this drawback, a series of switching filters, combining noise detection followed by noise reduction over the noise detected, have been studied in [4]-[9]. Also, techniques using fuzzy logic have been studied to solve this problem [10]-[11], and fuzzy metrics have shown to perform appropriately for this task [6, 7, 12, 13, 14, 15]. These works have proved that fuzzy logic and fuzzy metrics are appropriate for image denoising because it can deal with the nonlinear nature of digital images and with the inherent uncertainty in distinguishing between noise and image structures.

In this paper, we aim to point out that, apart from the particular filtering method, it is very important to appropriate choose the metric used within the filter. To do so, using the same filtering procedure, we present a study of the performance of different metrics, including recent fuzzy metrics and a novel fuzzy metric specifically designed to detect impulses.

The paper is structured as follows. Section 2 introduces the metrics used the detection process. The proposed study and experimental results are described in Section 3 with a performance comparison and discussion. Finally, some conclusions are drawn in Section 4.

2 Metrics to Diagnose Noise

In Mathematics, a metric is a function which defines a distance between elements of a set. In colour image filtering every pixel of the image is an RGB component vector with integer values between 0 and 255. Then, metrics provides a way to assess de closeness degree between two pixels. L_1 and L_2 metrics were the first in the experiences, followed by angular distance between pixels, and a set of combinations with several metrics. In this work we are going to use four metrics (two classics and two fuzzy). The classical metrics are L_1 and L_2:

$$L_1(\mathbf{x}_i, \mathbf{x}_j) = \sum_{l=1}^{3} abs\{x_i(l) - x_j(l)\} \tag{9.1}$$

$$L_2(\mathbf{x}_i, \mathbf{x}_j) = \sqrt{\sum_{l=1}^{3} (x_i(l) - x_j(l))^2} \tag{9.2}$$

A theory with an important grown in recent years has been fuzzy logic, due to its important use in control systems, expert systems, sensors in electronic devices, etc. At the same time, fuzzy topology and fuzzy metrics were deployed. For this reason, fuzzy metrics penetrate in the image denoising area with very good results. Recent works shown that the use of fuzzy metrics can improve the filtering method.

A stationary fuzzy metric [17]-[19], M, on a set X, is a fuzzy set of $X \times X$ satisfying the following conditions for all $x, y, z \in X$:

(FM1) $M(x, y) > 0$

(FM2) $M(x, y) = 1$ if and only if $x = y$

(FM3) $M(x, y) = M(y, x)$

(FM4) $M(x, z) \geq M(x, y) * M(y, z)$,

where $*$ is a continuous t-norm.

$M(x, y)$ represents the degree of nearness of x and y and, according to (FM2), $M(x, y)$ is close to 0 when x is far from y.

Let $(x_i(1), x_i(2), x_i(3))$ the colour image vector $\mathbf{x}_i$ in the RGB colour space, and let X the set $\{0, 1, \ldots, 255\}^3$ and fixed $K > 0$. Then, accord to [12, 16], the function $M : X \times X \rightarrow]0, 1]$ given by

$$M_*(\mathbf{x}_i, \mathbf{x}_j) = \prod_{l=1}^{3} \frac{min\{x_i(l), x_j(l)\} + K}{max\{x_i(l), x_j(l)\} + K} \tag{9.3}$$

is a stationary fuzzy metric, for the usual product, on X in the sense of George and Veeramani [18]. In this way, from now on $M_*(\mathbf{x}_i, \mathbf{x}_j)$ will be the fuzzy distance between the colour image vectors $\mathbf{x}_i$ and $\mathbf{x}_j$. Obviously M_* is M-bounded and it satisfies

$$0 < \left(\frac{K}{255 + K}\right)^3 \leq M_*(\mathbf{x}_i, \mathbf{x}_j) \leq 1 \tag{9.4}$$

for all $\mathbf{x}_i, \mathbf{x}_j \in X$.

We define the fuzzy set M_∞ on X^3 by

$$M_\infty(\mathbf{x}_i, \mathbf{x}_j) = \min_{l=1}^{3} \frac{\min\{x_i(l), x_j(l)\} + K}{\max\{x_i(l), x_j(l)\} + K} \tag{9.5}$$

M_∞ is a (stationary) fuzzy metric in the sense of George and Veeramani [18]. From the mathematical point of view the stationary fuzzy metric M_∞, started in [8], can be seen as a fuzzy version of the L_∞ classical metric and, like we will prove, it is especially sensitive to impulse noise.

These fuzzy metrics are non-uniform in the sense that the measure given for two different pairs of consecutive numbers (or vectors) may not be the same. In this way, increasing the value of K reduces this non-uniformity. According to our experiences, we have set $K = 1024$ which is an appropriate value for RGB colour vectors [12, 13].

(a) (b) (c) (d)

Figure 1: Test images: (a) Pills, (b) detail of Pills, (c) Statue, and (d) detail of Statue.

3 Experimental Study and Results

In recent works about image filtering, one of the most studied concerns impulse noise detection. The key issue is to distinguish between edges, fine details and noise. One switching method that provides good results is the Peer Group Filter (PGF), presented in Smolka [5]. This method provides a fast schema of noise detection and a posterior operation of noise replacement. In the first phase, the algorithm makes a study of the neighborhood of every pixel in a filtering window (of usual size 3×3), and if the pixel in study have at least m pixels close to it (we have chosen $m = 2$ as in [5]), the method detects this pixel as noisy free and as noisy otherwise. In the second phase, the noisy pixels are replaced with the output of the Arithmetic Mean Filter of the colour pixels in the neighborhood. The algorithm makes a study of the neighborhood of every pixel in a filtering window (size 3×3). If the pixel have at least m pixels close to it ($m = 2$), this pixel is diagnosed as noisy-free and as noisy otherwise. Lately, only the noisy pixels are replaced with the output of the Arithmetic Mean Filter of the colour pixels in the neighborhood.

To show the importance of the choice of the metric used to measure the distance or similarity between colour image pixels, we have implemented different versions of the PGF using four different metrics. We have chosen the city-block and Euclidean classical metrics and the M_* and M_∞ fuzzy metrics introduced in Section 2.

Two images (figure 1) have been corrupted with impulsive noise according to the model proposed by Plataniotis [2], and then filtered with the four different variants of the filter. To assess the performance, the Mean Absolute Error (MAE), Peak Signal to Noise Ratio (PSNR) and Normalized Colour Difference (NCD), have been used. Notice that for MAE and NCD lower values denote better performance, whereas PSNR is better for higher values.

The images in figure 1 have been corrupted by the impulsive noise model proposed by [2] before filtering them.

Tables 9.1-9.2 show the performance results of the metrics, whereas figure 4 show a graphical analysis from NCD, that is a reference measure that denotes the visual quality of the filtered image.

From the tables 9.1 and 9.2 we may conclude that the L_2 metric and the M_∞ fuzzy metric exhibit a much better performance than the rest, specially in terms of PSNR. What? is the question that a curious student must do itself. The reason is than the *square* and the *min* operation makes, are specially sensitive to the presence of impulse noise. In particular, the best results of all with M_∞ fuzzy metric provide improvements about 40% in MAE respect to L_1 and M_* and pretty goods respect L_2, specially when noisy intensity grows. We can see that when impulse noise affected at least one component of either $\mathbf{x}_i$ or $\mathbf{x}_j$, it would be

Table 9.1: Experimental results for the PGF Filter in the comparison with diverse metrics when filtering the Pills detail image corrupted with different densities p of fixed-value impulse noise.

p	0.05			0.10			0.15			0.20		
Metric	MAE	PSNR	NCD $\times 10^{-2}$	MAE	PSNR	NCD $\times 10^{-2}$	MAE	PSNR	NCD $\times 10^{-2}$	MAE	PSNR	NCD $\times 10^{-2}$
Noisy	2.31	22.40	3.52	4.99	19.14	7.16	7.04	17.71	10.41	9.47	16.34	14.38
L_1	1.48	30.65	0.84	2.54	28.52	1.44	3.03	27.67	2.44	5.66	24.15	3.80
L_2	0.86	33.03	0.66	2.01	29.51	1.22	3.21	27.43	2.09	4.64	25.40	3.17
M_*	1.33	31.17	0.81	2.27	29.02	1.42	3.04	27.61	2.48	5.45	24.29	3.73
M_∞	0.74	33.82	0.49	1.76	30.22	1.03	2.34	29.01	1.67	3.39	26.41	2.83

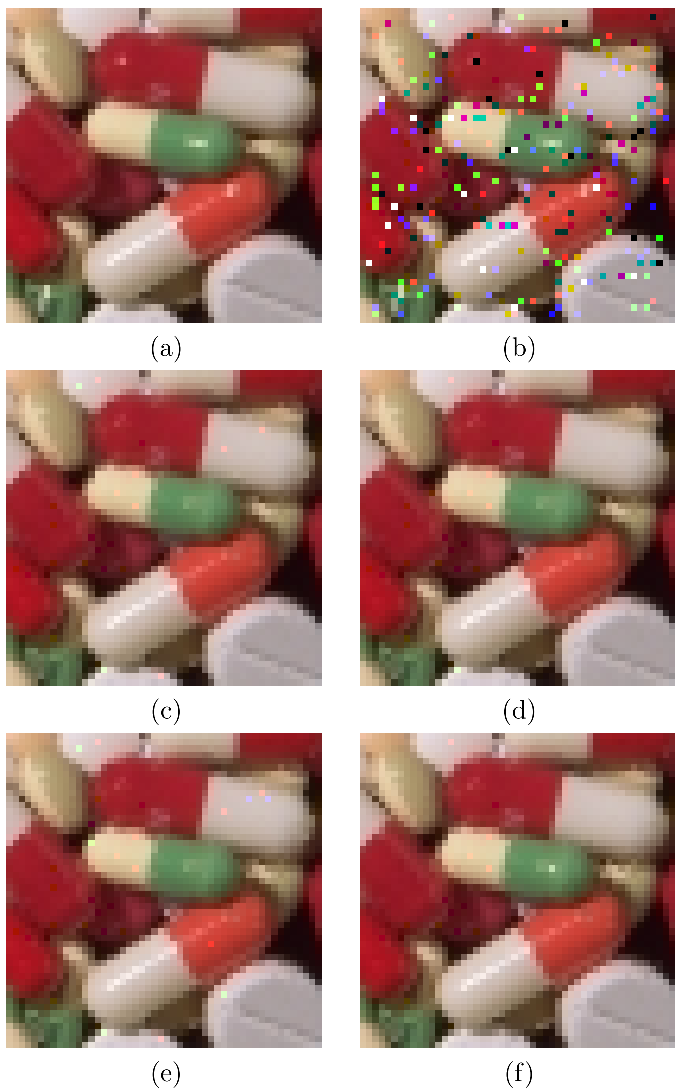
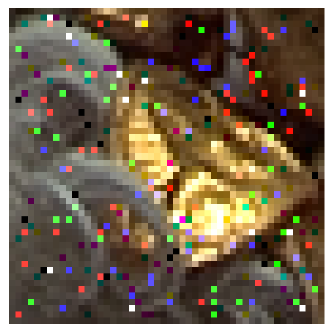


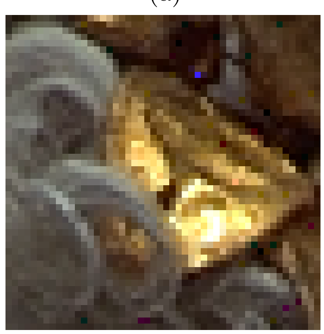
Figure 2: Visual comparison of the filter output using the Pills image and several metrics: (a) corrupted with $p = 10\%$ of impulsive noise, (b) L_1, (c) L_2, (d) M_* and (e) M_∞.

Table 9.2: Experimental results for the PGF Filter in the comparison with diverse metrics when filtering the Statue image corrupted with different densities p of fixed-value impulse noise.

p	0.05			0.10			0.15			0.20		
Metric	MAE	PSNR	NCD $\times 10^{-2}$	MAE	PSNR	NCD $\times 10^{-2}$	MAE	PSNR	NCD $\times 10^{-2}$	MAE	PSNR	NCD $\times 10^{-2}$
Noisy	1.93	23.37	3.41	5.14	19.03	8.99	7.16	17.44	12.52	9.20	16.35	15.74
L_1	2.76	25.51	2.30	5.09	22.85	3.42	6.84	21.94	4.56	7.80	21.14	5.64
L_2	2.56	25.79	1.89	4.07	24.14	2.70	5.10	23.42	3.60	6.43	22.01	4.90
M_*	1.40	28.48	2.11	3.74	24.68	3.20	6.47	22.31	4.29	7.65	21.31	5.44
M_∞	1.70	27.45	1.54	3.27	25.06	2.29	4.05	24.41	3.20	5.42	22.99	4.29


(a)


(b)


(c)


(d)


(e)


(f)

Figure 3: Visual comparison of the filter output using the Statue image and several metrics: (a) Original, (b) corrupted with $p = 10\%$ of impulsive noise, (c) L_1, (d) L_2, (e) M_* and (f) M_∞.

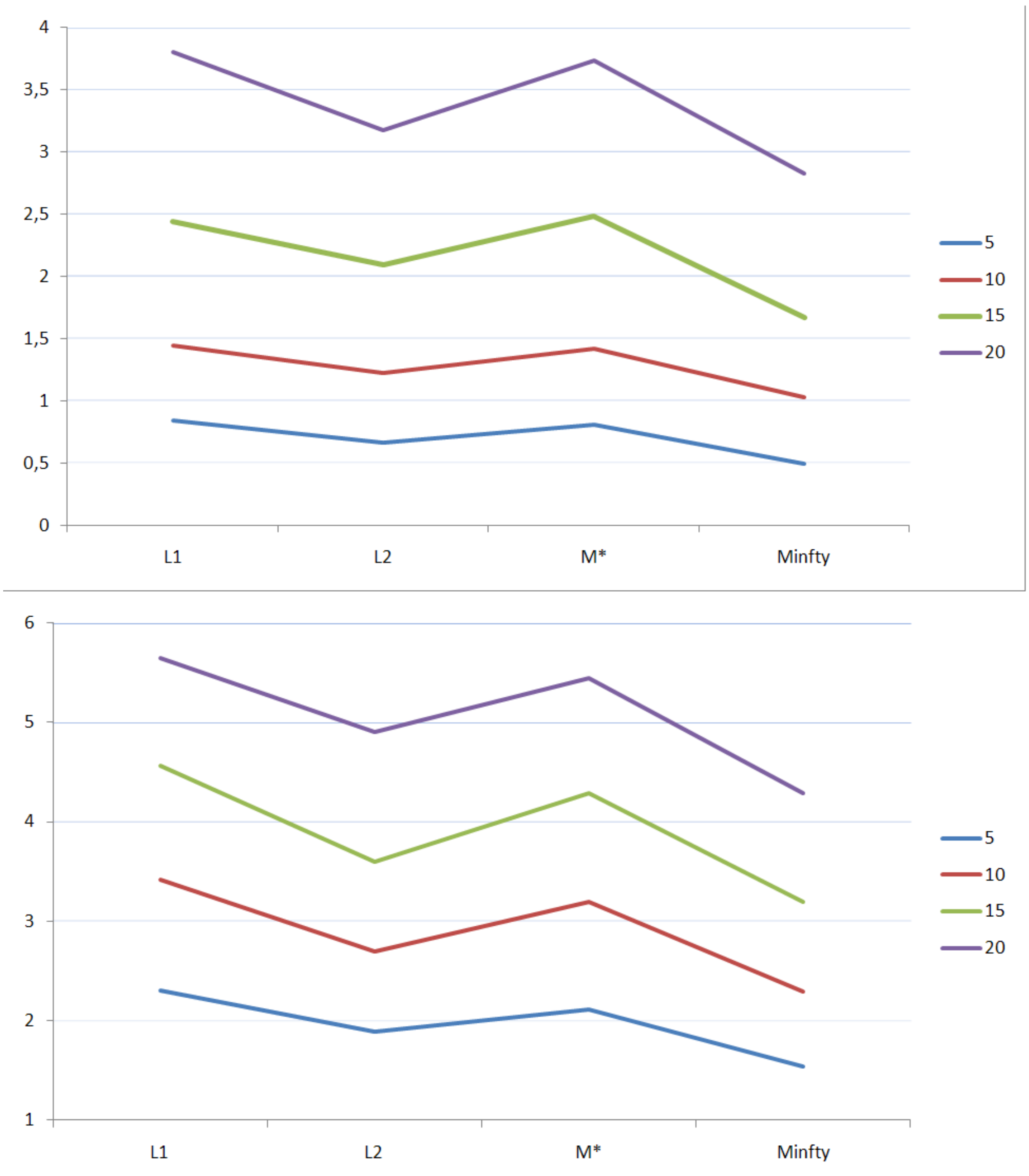


Figure 4: NCD performance varying metrics in Pills (above) and Statue (below).

associated to the lowest nearness value between their components. In such a case, the M_∞ fuzzy metric takes the nearness value associated to the presence of the impulse and ignores any possible similarity between the rest of the components. Moreover, as the difference between the components becomes larger, the value of M_∞ drops rapidly.

Figure 2 show a visual comparison of the output of every implementation whereas figure 4 show a visual analysis of the behaviour in terms of NCD of every image with every implementation.

4 Conclusions

In this paper we try to show students how important is to choose and accurate metric to measure distances, in this case, colorometric distance between pixels of digital color images. We have proved that a set of recent fuzzy metrics have better behaviour in front of classical metrics. This fact must encourage students and novel investigators to test the new mathematical tools (instead to use only the classical ones). Only varying the metric and filtering the images, the results obtained show that an appropriate choice of the metric is of paramount importance in the design of a filtering method. This choice can lead the filtering to significant performance benefits.

In this way is interesting to keep looking for new metrics and measures to improve the detection of noisy pixels, distinguishing them from edges and fine details contained in the images.

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